# PAPR Reduction of Wavelet-OFDM Signals Using Exponential Companding in Visible Light Communications

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Abstract-Visible light communication (VLC) is considered one of the newest wireless communication technologies that provides transmitted signals with high data rate. Orthogonal frequency division multiplexing (OFDM) is used extensively in VLC due to its ability to support high speed transmission and it gets ride of the inter-symbol interference [1]. OFDM enhancement with wavelet based on discrete wavelet transform (DWT) has achieved remarkable performance compared to OFDM due to interesting wavelet characteristics. OFDM suffers from the issue of peak to average power ratio (PAPR) which reduces the system efficiency especially in VLC system due to the nonlinearity of light emitting diode (LED) which distorts the signal. Wavelet proved an improvement in PAPR reduction of the radiated signal. In our study, an exponential companding PAPR reduction technique is applied on a wavelet-OFDM which satisfies better enhancement in PAPR reduction of the transmitted signal.

## Keywords—DWT; OFDM; PAPR; exponential companding; µlaw companding.

## I. INTRODUCTION

VLC has been widely used because it is an attractive alternative to the radio frequency (RF) system that has a small bandwidth in comparison to VLC. VLC has a very large scale bandwidth, low-cost end devices, free license operation, etc. [2].

OFDM is generally used in RF systems but it must be adapted for applications in VLC. The transmitted signal in VLC system must be unipolar to control the intensity of the LED. Also, the received optical signal must be real valued so that the photodetector can recover it. There are two common techniques to achieve these requirements. The first technique is DC-biased Optical OFDM (DCO-OFDM) where the input is first constrained to have a Hermitian symmetry then apply Inverse Fast Fourier Transform (IFFT) to get a real-valued signal, then apply a DC-bias to deliver a positive signal to the LED [3]- [4]. The second technique is Asymmetric Clipped Optical OFDM (ACO-OFDM) which is mainly used to avoid wasting power due to adding DC-bias in order to enhance the energy efficiency [5] - [6]. ACO-OFDM transmits the signal on the odd subcarriers so only half of the bandwidth is used [7].

Wavelet-OFDM has achieved a remarkable improvement in OFDM, in terms of bit error rate (BER) and PAPR on different channels such as an Additive White Gaussian Noise (AWGN) channel, a flat fading channel and a frequency selective channel [8].

Many comapanding techniques have been suggested for reducing the PAPR of RF baseband signal such as  $\mu$ -law companding [9], error function companding [10], and exponential companding [11]. The main problem in  $\mu$ -law companding technique that it only concentrates on increasing small signals, therefore growing the average power.

In this paper, we study the performance of applying exponential companding technique on a wavelet-OFDM and compare it with  $\mu$ -law companding in terms of PAPR and BER.

The remainder of this paper is organized as follows. In section II, Wavelet-OFDM based on DWT is described. In section III, µ-law and exponential companding techniques are described. In section IV, Illustrating LOS propagation model. In section V, the proposed algorithm of the exponential companding wavelet-OFDM (WOFDM) technique is demonstrated. Section VI gives simulation results for the proposed algorithm in LOS and AWGN channel corresponding to Complementary Cumulative Distribution Function (CCDF) of PAPR and BER. Finally, Section VII summarizes the paper.

## II. WAVELET-OFDM SYSTEM BASED ON DWT

## A. Discrete Wavelet Transform

Wavelet transform uses a scaled and translated versions of mother wavelet for signal analysis. Wavelet transform has several characteristics that make it very efficient for use in Multi Carrier Modulation (MCM) systems. Wavelet has a precise localization in both time and frequency domains [12]-[13]. Wavelet-OFDM removes the need for cyclic prefix then maintains the bandwidth and reduces the transmission power wastage [14]. Also, side lobes of modulated subcarriers in wavelet-OFDM are much lower than of original OFDM. DWT analyzes a limited time domain signal by decomposing it into two parts: the detail and approximation information. An efficient way to accomplish this scheme is by using filters where the approximation coefficients are the components of the low frequencies in the signal while the details coefficients are the components of the high frequencies in the signal. The most basic level of this filtering process is described in Fig. 1.



Fig. 1. The original signal is analyzed into two parts using high-pass and low-pass filters.

## B. OFDM based Discrete Wavelet Transform

In DWT-OFDM system, the transmitter uses digital modulator which converts serial data into OFDM symbols within a data stream of size N. Then each OFDM symbol  $X_{m(i)}$  is modulated in three steps. First, OFDM symbol is converted to serial in the form of a vector XX. Then, this vector is transposed into approximation coefficients (CA) by inverting the sign of the imaginary part and changing it from serial to parallel. Finally, the signal is filtered by LPF coefficients whereas zero padding signals  $0_{(N-1)}$  are transposed into detailed coefficients (CD) and filtered by HPF coefficients as in Fig. 2.



Fig. 2. The structure model of DWT- OFDM transmitter [15].

At the receiver side, the data Uk is filtered by high-pass and low-pass filters resulting detailed (CD) and approximated (CA) coefficients respectively. Then, the CA signal is demodulated by the digital modulator and the CD signal is discarded.



Fig. 3. The structure model of DWT- OFDM receiver [15].

#### III. COMPANDING TECHNIQUES FOR PAPR REDUCTION

## *A.* μ-law companding technique

Companding technique is a combination between compression and expansion. Compression function is used in the transmitter for increasing the value of small amplitude signals and compressing the large amplitude signals to protect small signals from noise and decrease the range of the transmitted signals. In the receiver, expansion function is inserted into the receiver model to oppose the change due to compression function.

In  $\mu$ -law companding technique, the compressor focuses on amplifying small signals. The characteristic equation of the  $\mu$ -law compressor is specified as [16]

$$y = \frac{V \ln \left(1 + \mu \frac{|x|}{V}\right)}{\ln(1 + \mu)} \operatorname{sgn}(x)$$
(1)

Where y represents the output signal, x is the input signal,  $\mu$  is the parameter of the compander, v is the highest value of the input signal x, and sgn(x) is the sign function that gets the sign of a real input number.  $\mu$ -law expander characteristic equation is given by

$$x = \frac{V}{\mu} \left( e^{\frac{|z|\ln(1+\mu)}{V}} - 1 \right) sgn(y)$$
(2)

## B. Exponential companding technique

Exponential companding is a nonlinear companding technique that expands small input signals and compress large input signals while retaining the average power unchanged. Therefore, the output signal is uniformly distributed.

For input signal x, the companding function is

$$f(x) = sgn(x) \sqrt{m \left[1 - exp\left(-\frac{x^2}{\sigma^2}\right)\right]}$$
(3)

Where m is a positive constant that specifies the average power of output signals and  $\sigma^2$  refers to the variance of the input signal. To keep the average power level unchanged, the value of m should be

$$m = \frac{E\{|S_n|^2\}}{E\{1 - exp(-\frac{|S_n|^2}{\sigma^2})\}}$$
(4)

Where E denotes the expectation and  $|S_n|$  is the amplitude of OFDM signal  $S_n$  that is specified by

$$|S_n| = \sqrt{Re^2 \{S_n\} + Im^2 \{S_n\}}$$
(5)

At the receiver side, the inverse of the companded function is

$$f^{-1}(x) = sgn(x) \sqrt{-\sigma^2 \ln(1 - \frac{x^2}{m})}$$
 (6)

## IV. LINE OF SIGHT (LOS) PROPAGATION MODEL

This model can be used when the transmitter is directly connected to the receiver without no obstacles. The transmitter is located at a specified height (H) with a horizontal inclination of  $\emptyset$ . The receiver is mounted with a zero vertical inclination ( $\theta$ =0). There is a horizontal space X between the transmitter and the receiver. The characteristic equation of the radiation beam is represented as

$$RE(\varphi,m) = \frac{m+1}{2\pi} \cdot P_T \cdot cos^m(\emptyset) \tag{7}$$

Where  $\varphi$  is the direction angle corresponding to the transmitter normal axis with  $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , P<sub>T</sub> is the output power, m is the mode number related to the radiation lobe that determines the direction of the source. It is specified as

$$m = -\frac{\ln(2)}{\ln(\cos(hpa))} \tag{8}$$

Where hpa denotes half power angle, which refers to the viewing angle at which 50% of the maximum radiant energy of the lobe. The signal intensity at the receiver is specified as

$$H(0) = \frac{m+1}{2\pi} \left( \frac{\cos m \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{H}{X}\right) - \emptyset\right) \cdot \cos \left(\tan^{-1} \left(\frac{X}{H}\right) - \emptyset\right)}{H^2 + X^2} \right) (9)$$

The received optical signal of a VLC link is given by

$$OPT_{RX} = H(0). OPT_{TX} + AWGN$$
(10)

Where  $OPT_{TX}$  is the transmitted optical signal.

## V. PROPOSED EXPONENTIAL-WOFDM SYSTEM

We propose in this section the effect of applying exponential companding techniques on wavelet-OFDM and study the performance of this system corresponding to BER and PAPR. The structure model of the proposed system is shown in Fig. 3. The input data is delivered to BPSK modulator and processed by Inverse Discrete Wavelet Transform (IDWT). After IDWT, we perform exponential companding technique. Therefore, the compressed OFDM signal is transmitted using LEDs. We simulate this algorithm using LOS and AWGN channels. Photo detector is used at the receiver to detect the transmitted light wave and transform it into electrical signal. The inverse of the exponential companding is applied to the signal then perform DWT process. Finally, the DWT signal is demodulated using BPSK demodulator to obtain the original data.



Fig. 4. Exponential companding-WOFDM system

#### VI. SIMULATION RESULTS AND DISCUSSION

In this section we simulate the performance of our proposed technique using the system parameters shown in table I.

Parameter	Value
Modulation scheme	BPSK
Wavelet used	Sym8
Channel model	LOS model + AWGN
SNR range	0 to 25 dB
Number of samples	10000
µ-value	255
Room (length, width, height)	10 x 10 x 5 m <sup>3</sup>
Height of transmitter (H)	5 m
LED launched power	120 mW
Minimum LED current	10 mA
Maximum LED current	2 A
LED cutoff frequency	20 MHz
<b>Receiver Half angle FOV</b>	60 (deg)
Active area of photodiode	$0.785 \text{ cm}^2$
Responsivity of photodiode	1 A/W

Table I. System Parameters

PAPR for DWT-OFDM, exponential-WOFDM, and  $\mu$ -law companded DWT-OFDM signals is simulated using matlab.

Fig. 5, shows the exponential-WOFDM signal has lower PAPR value than  $\mu$ -law companded and original DWT-OFDM signals.



Fig. 5. Relation between CCDF and PAPR values of DWT-OFDM, µ-law DWT, and exponential DWT signals.

The numerical results presented in table II show that there is a little difference in PAPR values between exponential-WOFDM and  $\mu$ -law companded DWT-OFDM. Also, there is a remarkable difference between DWT-OFDM and exponential-WOFDM. As a result, the proposed technique proved an improvement in terms of PAPR.

Table II. Numerical results of PAPR in DWT-OFDM techniques with different companding algorithms.

	Exponential-WOFDM	µ-law DWT-OFDM	DWT-OFDM
PAPR (dB)	1.1	1.253	3.479

In addition, we have studied the performance of DWT-OFDM, exponential-WOFDM, and DWT-OFDM with  $\mu$ -law companding in terms of BER. The BER performance as a function of SNR is examined for a LOS and AWGN channel.

Fig. 6, demonstrates the comparison of BER performances. It is clear that exponential-WOFDM system has lower SNR values than  $\mu$ -law companded signal at the same BER whereas it has larger SNR values in comparison with DWT-OFDM signal.



Fig. 6. BER for DWT-OFDM,  $\mu$ -law companded DWT, and exponential companded DWT signals.

It is clear that exponential-WOFDM system has 5.981 dB gain in terms of SNR values at BER= $10^{-4}$  over the  $\mu$ -law companded DWT-OFDM system. Also, it is shown that, DWT-OFDM system has SNR value lower than exponential-WOFDM system with 1.931 dB. The numerical results are presented in Table III.

Table III. Numerical results of SNR at  ${\rm BER}{=}10^{\text{-}4}$  in DWT-OFDM techniques with different companding algorithms.

	DWT-OFDM	Exponential-WOFDM	µ-law DWT-OFDM
SNR (dB)	9.51	11.441	17.422

For more analysis for the proposed algorithm, we studied the performance of different wavelets in DWT-OFDM, exponential-WOFDM, and  $\mu$ -law companded DWT-OFDM signals as shown in fig. 7.



Fig. 7. Relation between CCDF and PAPR values of DWT-OFDM, μ-law DWT, and exponential DWT signals for different wavelets.

Simulation using different wavelets in terms of PAPR leads to many results as shown in table IV. The lowest PAPR values in  $\mu$ -law DWT-OFDM and DWT-OFDM are accomplished using sym3 wavelet that are 1.045 dB and 2.809 dB respectively. The best result is the minimum PAPR value that is achieved when using the proposed algorithm with db8 wavelet.

Table IV. Numerical results of PAPR (dB) in DWT-OFDM,  $\mu$ -law companded DWT, and exponential companded DWT signals for different wavelets.

	Exponential-WOFDM	µ-law DWT-OFDM	DWT-OFDM
Sym8	0.921	1.161	3.479
Sym3	1.045	1.045	2.809
db8	0.5325	1.435	5.709

BER analysis using different wavelets for DWT-OFDM,  $\mu$ -law DWT-OFDM, and exponential-WOFDM is demonstrated in fig. 8.



Fig. 8. BER for DWT-OFDM,  $\mu$ -law companded DWT, and exponential companded DWT signals for different wavelets.

There is no change in the SNR value at DWT-OFDM when using different wavelets. DWT-OFDM has the lowest SNR value at BER=10-4 as shown in table V.

Table V. Numerical results of SNR in dB at  $BER=10^4$  in DWT-OFDM,  $\mu$ -law companded DWT, and exponential companded DWT signals for different wavelets.

	Exponential-WOFDM	µ-law DWT-OFDM	DWT-OFDM
Sym8	11.513	18	9.521
Sym3	12	18	9.521
db8	15	20.112	9.521

## VII. CONCLUSIONS

DWT-OFDM with exponential companding technique has offered better enhancement in terms of BER and PAPR than DWT-OFDM with  $\mu$ -law companding. In comparison of ordinary DWT-OFDM and DWT-OFDM with exponential companding, the PAPR in DWT-OFDM with exponential companding has superior improvement than ordinary DWT-OFDM. In terms of BER, ordinary DWT-OFDM has lower BER value than DWT-OFDM with exponential companding. Companding techniques have no restriction on modulation format and subcarrier size in addition to low implementation complexity.

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